

BRST COHOMOLOGY OF THE SUPERSTRING IN SUPER-BELTRAMI PARAMETRIZATION

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Abstract

A method for the calculation of the BRST cohomology, recently developed for 2D gravity theory and the bosonic string in Beltrami parametrization, is generalized to the superstring theories quantized in super-Beltrami parametrization.

1 Introduction

The BRST method for quantization of general theories is the most important method of quantization and it has been applied to all gauge theories [5]. In this framework the search for the invariant Lagrangian, the anomalies and the Schwinger terms corresponding to a given set of field transformations can be done in a purely algebraic way by solving the BRST consistency condition in the space of the integrated local field polynomials [6, 8, 9, 10]. This amounts to study the nontrivial solutions of the equation

$$s\mathcal{A} = 0 \tag{1.1}$$

with s the nilpotent BRST differential and \mathcal{A} and integrated local functional $\mathcal{A} = \int d^2x f$. The condition (1.1) translates into the local descent equations [8]:

$$\begin{aligned} s\omega_2 + d\omega_1 &= 0, & s\omega_1 + d\omega_0 &= 0 \\ s\omega_0 &= 0 \end{aligned} \tag{1.2}$$

where ω_2 is a 2-form with $\mathcal{A} = \int \omega_2$ and ω_1, ω_0 are *local* 1- and 0-forms. It is well known [9, 10, 15] that the descent equations terminate in the bosonic string or the superstring in Beltrami or super-Beltrami parametrization, always with a nontrivial 0-form ω_2 and that their "integration" is trivial

$$\omega_1 = \delta\omega_0 \qquad \omega_2 = \frac{1}{2}\delta^2\omega_0 \tag{1.3}$$

where the operator δ was introduced by Sorella [8] and it allows to express the exterior derivative d as a BRST commutator

$$d = -[s, \delta]. \tag{1.4}$$

Thus it is sufficient to determine the general solution of

$$s\omega_0 = 0 \tag{1.5}$$

in the space of local functions of the fields and their derivatives, i.e. to calculate the BRST cohomology group $H(s)$.

In this paper we shall investigate the structure of $H(s)$ for the superstring theory in the super-Beltrami parametrization. The Beltrami superfield parametrization was introduced for the first time in physics by Gates and Nishino and independently by Rocek and al.[1]. Further studies of heterotic strings in Beltrami superfield formulation as well as 2D supergravity were made in [2, 3] and more recently in [4]. We shall also make use of the results for the superfield as well as for the component field formalism obtained in [12, 13, 11, 14, 15]. The investigation of (1.5) is considerable simplified by introducing an appropriate new basis of variables substituting the field and their derivatives. The way in which this new basis is chosen is a crucial step in the calculation.

The paper is organized as follows. In Sec.2 we briefly recall the BRST symmetry within the superstring in the super-Beltrami parametrization. In Sec.3 we split the algebra of fields and their derivatives \mathcal{A} in the contractive part \mathcal{C} and the minimal part \mathcal{M} by using the famous Sullivan Theorem [16]. In Sec.4 we introduce a new basis and show how we can calculate the elements of $H(s)$. At the end we shall give some explicit examples including the classical action S_0 and the anomalies.

2 BRST symmetry for the superstring

Let us start by introducing the setup for the superstring in the super-Beltrami parametrization. We shall work on a Riemann surface, i.e. a real, smooth 2-dimensional manifold with a positive definite metric and the local coordinates (x, y) or complex coordinates $(z = x + iy, \bar{z} = x - iy)$. The classical fields are $\{X, \lambda, \bar{\lambda}, F, \mu, \bar{\mu}, \alpha, \bar{\alpha}\}$ where $X = (X^\mu)$ are the string coordinates, $\lambda, \bar{\lambda}$ are their fermionic superpartners and F are auxiliary fields, μ stands for the bosonic Beltrami differential and α , called "Beltramino", is its fermionic superpartner. The doublets (μ, α) and $(\bar{\mu}, \bar{\alpha})$ characterize the super-Beltrami parametrization in the 2d-superspace. These variables describe the full (1,1) superstring theory. According to Delduc and Gieres [11], the (1,0) version can be obtained by truncation $\bar{\alpha} = \bar{\lambda} = F = 0$, i.e. in this case the supersymmetry is present only in one sector, the other being just the bosonic string.

We shall suppose that the theory is invariant under 2d superdiffeomorphism which is expressed by the following BRST transformations [11]:

$$s\Phi = c\Phi_{1,0} + \bar{c}\Phi_{0,1} + \frac{1}{2}\epsilon\Phi_\epsilon + \frac{1}{2}\bar{\epsilon}\Phi_{\bar{\epsilon}} + a_\Phi(\partial c)\Phi + b_\Phi(\bar{\partial}\bar{c})\Phi \quad (2.1)$$

with $\Phi = \{X, \lambda, \bar{\lambda}, F\}$ and

$$\Phi_{1,0} = D_z\Phi, \quad \Phi_{0,1} = D_{\bar{z}}\Phi \quad (2.2)$$

The supercovariant derivatives $D_z\Phi$ and $D_{\bar{z}}\Phi$ for $\Phi = \{X, \lambda, \bar{\lambda}, F\}$ are defined as in [11]:

$$D_z X = \frac{1}{1 - \mu\bar{\mu}}[(\partial - \bar{\mu}\bar{\partial})X + \frac{1}{2}\bar{\mu}\alpha\lambda - \frac{1}{2}\bar{\alpha}\bar{\lambda}] \quad (2.3)$$

$$D_z \lambda = \frac{1}{1 - \mu\bar{\mu}}[(\partial - \bar{\mu}\bar{\partial} + \frac{1}{2}\bar{\mu}\partial\mu)\lambda - \frac{1}{2}\bar{\mu}\alpha D_z X + \frac{i}{2}\bar{\alpha}F] \quad (2.4)$$

$$D_{\bar{z}} \lambda = \frac{1}{1 - \mu\bar{\mu}}[(\bar{\partial} - \mu\partial - \frac{1}{2}\partial\mu\lambda - \frac{1}{2}\alpha(D_z X) - \frac{i}{2}\mu\bar{\alpha}F] \quad (2.5)$$

$$D_z F = \frac{1}{1 - \mu\bar{\mu}}[(\partial - \bar{\mu}\bar{\partial} - \frac{1}{2}\bar{\partial}\bar{\mu} + \frac{1}{2}\bar{\mu}\partial\mu)F - \frac{i}{2}\bar{\alpha}(D_{\bar{z}}\lambda) - \frac{i}{2}\bar{\mu}\alpha(D_z\bar{\lambda})] \quad (2.6)$$

and their complex conjugates, which are obtained by putting bars everywhere and replacing ' i ' by ' $-i$ '. In (2.6) we have used the notations $\partial = \partial_z = \partial/\partial z$ and $\bar{\partial} = \partial_{\bar{z}} = \partial/\partial \bar{z}$. The fields Φ_ϵ and $\Phi_{\bar{\epsilon}}$ are

$$X_\epsilon = \lambda, \quad X_{\bar{\epsilon}} = \bar{\lambda} \quad (2.7)$$

$$\lambda_\epsilon = X^{1,0}, \quad \lambda_{\bar{\epsilon}} = -iF \quad (2.8)$$

$$\bar{\lambda}_\epsilon = iF, \quad \bar{\lambda}_{\bar{\epsilon}} = X^{0,1} \quad (2.9)$$

$$F_\epsilon = -i\bar{\lambda}^{1,0}, \quad F_{\bar{\epsilon}} = i\lambda^{1,0} \quad (2.10)$$

The numerical coefficients a_Φ and b_Φ , which will play a crucial role in the following, have the values:

$$\begin{aligned} a_X &= b_X = 0, & a_F &= \frac{1}{2}, b_F = \frac{1}{2} \\ a_\lambda &= \frac{1}{2}, b_\lambda = 0, & a_{\bar{\lambda}} &= 0, b_{\bar{\lambda}} = \frac{1}{2}. \end{aligned} \quad (2.11)$$

The pairs (a_Φ, b_Φ) is, by definition, *the total weight* of the field Φ . The BRST transformations of the super-Beltrami differentials (μ, α) are given by:

$$s\mu = \bar{\partial}c - \mu\partial c + (\partial\mu)c + \frac{1}{2}\alpha\epsilon \quad (2.12)$$

$$s\alpha = \bar{\partial}\epsilon - \mu\partial\epsilon + \frac{1}{2}(\partial\mu)\epsilon + c(\partial\alpha) + \frac{1}{2}\alpha\partial c. \quad (2.13)$$

and their c.c. Finally the BRST transformations of the ghosts (c, \bar{c}) and $(\epsilon, \bar{\epsilon})$ are:

$$sc = c\partial c - \frac{1}{4}\epsilon\epsilon \quad (2.14)$$

$$s\epsilon = c\partial\epsilon - \frac{1}{2}\epsilon\partial c \quad (2.15)$$

and their c.c. so that

$$s^2 = 0. \quad (2.16)$$

The fermionic ghosts (c, \bar{c}) and their *bosonic* superpartners $(\epsilon, \bar{\epsilon})$ are related to the usual superdiffeomorphism ghosts (ξ, \bar{x}) and $(\xi^\theta, \bar{\xi}^\theta)$ by [11]

$$c = \xi + \mu\bar{\xi}, \quad \epsilon = \xi + \bar{\xi}\alpha, \quad (2.17)$$

and their c.c.

For the superstring in the super-Beltrami parametrization the operator δ , introduced by Sorella [8, 9, 15], which satisfies eq.(1.4) is defined by

$$\begin{aligned} \delta c &= dz + \mu d\bar{z} & , & & \delta \bar{c} &= d\bar{z} + \bar{\mu} dz \\ \delta \epsilon &= -\alpha d\bar{z} & , & & \delta \bar{\epsilon} &= -\bar{\alpha} dz \\ \delta \Phi &= 0 & \text{for} & & \Phi &= \{X, \lambda, \bar{\lambda}, F, \mu, \bar{\mu}, \alpha, \bar{\alpha}\}. \end{aligned} \quad (2.18)$$

The operator δ is of total degree 0 and obeys the following relations

$$d = -[s, \delta] \quad , \quad [d, \delta] = 0. \quad (2.19)$$

The operator δ can be used to solve the descent equations (1.2). As it has been shown in [17, 18, 19] these solutions can be obtained from the equation

$$(s + d)\omega_0(c + dz + \mu d\bar{z}, \bar{c} + d\bar{z} + \bar{\mu} dz, \epsilon - \alpha d\bar{z}, \bar{\epsilon} - \bar{\alpha} dz, X, \lambda, \bar{\lambda}, F, \mu, \bar{\mu}, \alpha, \bar{\alpha}) = 0 \quad (2.20)$$

where ω_0 is a solution of the equation (1.5), by projecting out different terms with a given ghost and space-time degree.

The main purpose of our paper is to solve the equation (1.5) in the algebra of all local polynomials of the fields \mathcal{A} . A basis for this algebra can be chosen to consist of

$$\left\{ \partial^p \bar{\partial}^q \Psi, \partial^p \bar{\partial}^q c, \partial^p \bar{\partial}^q \bar{c}, \partial^p \bar{\partial}^q \epsilon, \partial^p \bar{\partial}^q \bar{\epsilon} \right\}, \quad (2.21)$$

where $\Psi = \{X, \lambda, \bar{\lambda}, F, \mu, \bar{\mu}, \alpha, \bar{\alpha}\}$. However, the BRST transformations of this basis is quite complicated and it contains many terms which can be eliminated in $H(s)$. In the next section we shall eliminate a part of the basis (2.21) and in the following one we shall introduce a new basis, where the action of the BRST differential s is very simple.

3 Contractive algebra and Sullivan Theorem

The calculation of the BRST cohomology ,i.e. the solutions of the equations

$$s\omega_0 = 0 \quad (3.1)$$

can be considerably simplified if we organize the algebra \mathcal{A} as a free differentiable algebra and use a very strong and efficient theorem due to D.Sullivan [16]. A free differential algebra is an algebra generated by a basis endowed with a differential. Sullivan's tells us that the most general free differential algebra \mathcal{A} is a tensor product of a contractible algebra \mathcal{C} and a minimal one \mathcal{M} . A minimal differential algebra \mathcal{M} with the differential s is an algebra for which $\mathcal{M} \subseteq \mathcal{M}^+ \mathcal{M}^+$ where \mathcal{M}^+ is the part of \mathcal{M} in positive degree, i.e. $\mathcal{M} = \mathcal{C} \oplus \mathcal{M}^+$ and a contractive differential algebra \mathcal{C} is one isomorphic to the tensor product of those of the form $\Lambda(x, sx)$.

On the other hand, due to K  neth theorem the cohomology of \mathcal{A} is given by the cohomology of its minimal part \mathcal{M} and we can say that the contractible subalgebra \mathcal{C} can be neglected in the calculation of the cohomology $H(s)$. In [20, 21, 22] was given a general iterative construction of the minimal differential algebra of a given free differential algebra \mathcal{A} and this construction can be applied in our case. However, for our differential algebra the construction of \mathcal{C} and \mathcal{M} is straightforward and we do not have to rely on the general construction. In fact it is easy to see from (2.13) and (2.14) that the generators of (2.21) of \mathcal{A} which have the form

$$\{\partial^p \bar{\partial}^q (\bar{\partial} c) \ , \ \partial^p \bar{\partial}^q (\partial \bar{c}) \ , \ \partial^p \bar{\partial}^q (\bar{\partial} \epsilon) \ , \ \partial^p \bar{\partial}^q (\partial \bar{\epsilon})\} \quad (3.2)$$

with $p, q = 0, 1, \dots$ can be replaced by

$$\{\partial^p \bar{\partial}^q \Phi \ , \ s(\partial^p \bar{\partial}^q \Phi) \ , \ \partial^p c \ , \ \bar{\partial}^p \bar{c} \ , \ \partial^p \epsilon \ , \ \bar{\partial}^p \bar{\epsilon}\} \quad (3.3)$$

with $\Phi = \{\mu \ , \ \bar{\mu} \ , \ \alpha \ , \ \bar{\alpha}\}$ and the algebra \mathcal{A} could be generated by

$$\{\partial^p \bar{\partial}^q \Phi \ , \ s(\partial^p \bar{\partial}^q \Phi) \ , \ \partial^p \bar{\partial}^q (\Psi) \ , \ \partial^p c \ , \ \bar{\partial}^p \bar{c} \ , \ \partial^p \epsilon \ , \ \bar{\partial}^p \bar{\epsilon}\} \quad (3.4)$$

where $\Psi = (X \ , \ \lambda \ , \ \bar{\lambda} \ , \ F)$. Now the Sullivan decomposition can be easily obtained from (3.4) since the contractible algebra \mathcal{C} is generated by

$$\{\partial^p \bar{\partial}^q \Phi \ , \ s(\partial^p \bar{\partial}^q \Phi)\} \quad (3.5)$$

and the minimal subalgebra \mathcal{M} might be generated by the elements

$$\{\partial^p \bar{\partial}^q \Psi \ , \ \partial^p c \ , \ \bar{\partial}^p \bar{c} \ , \ \partial^p \epsilon \ , \ \bar{\partial}^p \bar{\epsilon}\}. \quad (3.6)$$

However, the basis (3.6) might not generate a minimal differential algebra since the derivatives of Ψ do not have simple BRST transformations and it is difficult to see whether the condition $\mathcal{M} \subseteq \mathcal{M}^+ \cdot \mathcal{M}^+$ is satisfied or not. The situation can be considerable improved if one introduces a new basis. To define this new basis we introduce four linear even operators, which will play a crucial role in our considerations:

$$\Delta = \{s \ , \ \frac{\partial}{\partial c}\} \ , \ \bar{\Delta} = \{s \ , \ \frac{\partial}{\partial \bar{c}}\} \quad (3.7)$$

$$\Delta_0 = \{s \ , \ \frac{\partial}{\partial(\partial c)}\} \ , \ \bar{\Delta}_0 = \{s \ , \ \frac{\partial}{\partial(\partial \bar{c})}\} \quad (3.8)$$

These operators are, in fact, even differentials since they satisfy Leibniz rule

$$D(ab) = (Da)b + a(Db)$$

where D is one of the operators just defined.

The operators (3.7) could replace the usual derivatives in the new basis defined by

$$\Phi^{p,q} = \Delta^p \bar{\Delta}^q \Phi \quad \text{where} \quad \Phi = \{X, \lambda, \bar{\lambda}, F\} \quad (3.9)$$

and

$$\begin{aligned} c^p &= \frac{1}{(p+1)!} \Delta^{p+1} c = \frac{1}{(p+1)!} \partial^{p+1} c, \quad \bar{c}^p = \frac{1}{(p+1)!} \bar{\Delta}^{p+1} \bar{c} = \frac{1}{(p+1)!} \bar{\partial}^{p+1} \bar{c} \\ \epsilon^{p+\frac{1}{2}} &= \frac{1}{2} \frac{1}{(p+1)!} \Delta^{p+1} \epsilon = \frac{1}{(p+1)!} \partial^{p+1} \epsilon, \quad \bar{\epsilon}^{p+\frac{1}{2}} = \frac{1}{2} \frac{1}{(p+1)!} \bar{\Delta}^{p+1} \bar{\epsilon} = \frac{1}{(p+1)!} \bar{\partial}^{p+1} \bar{\epsilon} \end{aligned} \quad (3.10)$$

where $p = -1, 0, +1, \dots$.

The action of the BRST differential s on the new basis can be easily obtained if we use the commutation relations

$$[\Delta, s] = 0, \quad [\bar{\Delta}, s] = 0 \quad (3.11)$$

Thus eqs. (2.1) and (3.11) yield

$$s\Phi^{p,q} = \sum_{k=-1} (c^k L_k + \bar{c}^k \bar{L}_k + \epsilon^{k+\frac{1}{2}} G_{k+\frac{1}{2}} + \bar{\epsilon}^{k+\frac{1}{2}} \bar{G}_{k+\frac{1}{2}}) \Phi^{p,q} \quad (3.12)$$

where

$$\begin{aligned} L_k \Phi^{p,q} &= A_k^p(a_\Phi) \Phi^{p-k,q}, & \bar{L}_k \Phi^{p,q} &= A_k^q(b_\Phi) \Phi^{p,q-k}, \\ G_{k+\frac{1}{2}} \Phi^{p,q} &= A_{k+\frac{1}{2}}^p(\Phi_\epsilon) \Phi^{p-k-\frac{1}{2},q}, & \bar{G}_{k+\frac{1}{2}} \Phi^{p,q} &= A_{k+\frac{1}{2}}^q(\Phi_\epsilon) \Phi^{p,q-k-\frac{1}{2}}, \end{aligned} \quad (3.13)$$

with

$$A_k^p(a) = \frac{p!}{(p-k)!} [p-k+a(k+1)], \quad A_k^p = \frac{p!}{(p-k)!}. \quad (3.14)$$

In eqs.(3.13) the fields $\Phi_\epsilon^{p,q}$ have the form given by eq. (3.9), i.e.

$$\Phi_\epsilon^{p,q} = \Delta^p \bar{\Delta}^q \Phi_\epsilon \quad (3.15)$$

and are given by eqs.(2.8), (2.9), (2.10), (2.10).

The linear operators $\{L_k, \bar{L}_k, G_{k+\frac{1}{2}}, \bar{G}_{k+\frac{1}{2}}\}$ represent on $\Phi^{p,q}$ the super-Virasoro algebra since they obey on $\Phi^{p,q}$ the (anti)commutation relations:

$$[L_m, L_n] = (m-n)L_{m+n}, \quad \{G_a, G_b\} = 2L_{a+b}, \quad [L_n, G_a] = (\frac{n}{2} - a)G_{a+b} \quad (3.16)$$

where $m, n = -1, 0, 1, \dots$ and $a, b = -\frac{1}{2}, \frac{1}{2}, \dots$ and the same relations with L_m and G_a replaced by \bar{L}_m and \bar{G}_a . In addition

$$[L_m, \bar{L}_n] = [L_m, \bar{G}_a] = [G_a, \bar{L}_m] = 0 \quad (3.17)$$

$$\{G_a, \bar{G}_b\} = 0 \quad (3.18)$$

Thus the generators of the BRST transformations on $\Phi^{p,q}$ form two copies of the super-Virasoro algebra without central charge. All these operators can be defined on the whole algebra by fields $\Phi^{p,q}$ as even or odd derivatives. In order to do that we just notice that from (3.13) one can write

$$L_k = \{s, \frac{\partial}{\partial c^k}\} \quad , \quad G_{k+\frac{1}{2}} = [\frac{\partial}{\partial \epsilon^{k+\frac{1}{2}}}, s] \quad (3.19)$$

and by taking into account that s and $\frac{\partial}{\partial c^k}$ are odd derivatives and $\frac{\partial}{\partial \epsilon^{k+\frac{1}{2}}}$ are even one can extend L_k and $G_{k+\frac{1}{2}}$ as even and odd derivatives, respectively.

The BRST transformations of the ghosts basis $\{c^n, \bar{c}^n, \epsilon^{n+\frac{1}{2}}, \bar{\epsilon}^{n+\frac{1}{2}}\}$ ($n = -1, 0, 1, \dots$) can easily be written as:

$$\begin{aligned} s c^n &= \frac{1}{2} f_{pq}^n c^q c^p - \frac{1}{2} \bar{f}_{ab}^n \epsilon^a \epsilon^b \\ s \epsilon^a &= -\tilde{f}_{mb}^a c^m \epsilon^b \end{aligned} \quad (3.20)$$

where

$$f_{pq}^n = (p - q) \delta_{p+q}^n \quad , \quad \bar{f}_{ab}^n = 2 \delta_{a+b}^n \quad , \quad \tilde{f}_{mb}^a = (\frac{m}{2} - b) \delta_{m+b}^a \quad (3.21)$$

and $p, q = -1, 0, \dots, a = -\frac{1}{2}, \frac{1}{2}, \dots$. The new basis (3.9) and (3.10) have been introduced by Brandt, Troost and Van Proeyen [23] for 2D-gravity and was used in [19] for Beltrami parametrization.

The main result of all these considerations is the fact that the subalgebra \mathcal{M} generated by

$$\{\Phi^{pq}, c^p, \bar{c}^p, \epsilon^p, \bar{\epsilon}^p\}$$

is a minimal algebra and its BRST cohomology coincide with $H(s)$. Therefore, at this stage to calculate the BRST cohomology $H(s)$ we simply have to consider only the minimal subalgebra \mathcal{M} together with its BRST transformations (3.12) and (3.20).

In fact one can reduce the number of the candidates for the members of $H(s)$ if we use the operators $\Delta_0, \bar{\Delta}_0$ introduced in (3.8). They have the remarkable property that all the elements of the basis of \mathcal{M} are their eigenfunctions:

$$\begin{aligned} \Delta_0 \Phi^{pq} &= (a_\Phi + p) \Phi^{pq} \quad , \quad \bar{\Delta}_0 \Phi^{pq} = (b_\Phi + q) \Phi^{pq} \\ \Delta_0 c^p &= p c^p \quad , \quad \bar{\Delta}_0 c^p = 0 \\ \Delta_0 \bar{c}^p &= 0 \quad , \quad \bar{\Delta}_0 \bar{c}^p = p \bar{c}^p \\ \Delta_0 \epsilon^a &= a \epsilon^a \quad , \quad \bar{\Delta}_0 \epsilon^a = 0 \\ \Delta_0 \bar{\epsilon}^a &= 0 \quad , \quad \bar{\Delta}_0 \bar{\epsilon}^a = a \bar{\epsilon}^a. \end{aligned} \quad (3.22)$$

where $p, q = -1, 0, 1, \dots, a = -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \dots$. Therefore all monomials from \mathcal{M} are eigenvalues of Δ_0 and $\bar{\Delta}_0$ and their eigenvalues are, by definition, *the total weight*. Due to the fact that Δ_0 and

$\bar{\Delta}_0$ are the anticommutators of s with something it is easy to convince ourselves that a solution of the equation $s\omega_0 = 0$ with a total weight different from $(0,0)$ is s -exact, i.e. if

$$s\omega_0 = 0 \quad , \quad \Delta_0\omega_0 = p\omega_0 \quad (p = 1, 2, \dots)$$

then

$$\omega_0 = s\left(\frac{1}{p}\frac{\partial\omega_0}{\partial(\partial c)}\right).$$

4 The new basis and BRST cohomology

There is an essential difference between the Beltrami and super-Beltrami parameterizations when one tries to calculate the BRST cohomology. For the 2D gravity Brandt, Troost and Van Proeyen [29] (see also [23, 28, 19]) have shown that the basis of the BRST cohomology $H(s)$, contains a finite number of terms with very simple forms. For the super-Beltrami parametrization this is not the case due to the presence of the bosonic ghosts ϵ and $\bar{\epsilon}$. In this case, even if we impose the condition of the weight to be $(0,0)$, we obtain a basis for the BRST cohomology group $H(s)$ with an infinite number of terms. Besides, the form of these terms is much more involved in comparison with the Beltrami parametrization. Therefore, we are not going to give a complete discussion but rather we shall analyze only the general solution of (3.1) for ghost number two and we give some particular solutions for ghost number three. They are in fact the supersymmetric extensions of the BRST cohomology elements for the non-supersymmetric case, that is, for the Beltrami parametrization [28, 19].

For ghost number two we consider the general solution of (3.1) of the form

$$\omega_0 = c\bar{c}\Pi_1 + \epsilon\bar{c}\Pi_2 + \bar{\epsilon}c\Pi_3 + \epsilon\bar{c}\Pi_4 + \epsilon c\Pi_5 + \epsilon\bar{c}\Pi_6 \quad (4.1)$$

the total weight of Π_j , ($j = 1, 2, 3, 4$) must have the values

$$w(\Pi_1) = (1, 1) \quad , \quad w(\Pi_2) = \left(\frac{1}{2}, 1\right) \quad , \quad w(\Pi_3) = \left(1, \frac{1}{2}\right) \quad , \quad w(\Pi_4) = \left(\frac{1}{2}, \frac{1}{2}\right)$$

and they are polynomials in the classical fields and their derivatives, not containing any ghost. Now we want to impose the conditions that ω_0 is a solution of eq.(3.1). Taking into consideration the BRST transformations of all ghosts (2.14) and (2.15) and for the classical fields (3.12) the equation (3.1) yields

$$L_k\Pi_A = \bar{L}_k\Pi_A = G_a\Pi_A = \bar{G}_a\Pi_A = 0 \quad (4.2)$$

with $k = 1, 2, \dots, a = \frac{3}{2}, \frac{5}{2}, \dots$ and $A = 1, 2, 3, 4$,

$$\begin{aligned} L_0\Pi_1 &= \Pi_1 \quad , \quad \bar{L}_0\Pi_1 = \Pi_1 \\ L_0\Pi_2 &= \frac{1}{2}\Pi_2 \quad , \quad \bar{L}_0\Pi_2 = \Pi_2 \\ L_0\Pi_3 &= \Pi_3 \quad , \quad \bar{L}_0\Pi_3 = \frac{1}{2}\Pi_3 \\ L_0\Pi_4 &= \frac{1}{2}\Pi_4 \quad , \quad \bar{L}_0\Pi_4 = -\frac{1}{2}\Pi_4 \end{aligned} \quad (4.3)$$

and

$$\begin{aligned}
\Pi_1 &= -G_{-\frac{1}{2}}\Pi_2, \quad \Pi_1 = -\bar{G}_{-\frac{1}{2}}\Pi_3 \\
\Pi_2 &= -G_{\frac{1}{2}}\Pi_1, \quad \Pi_3 = \bar{G}_{\frac{1}{2}}\Pi_1, \\
\Pi_4 &= -G_{\frac{1}{2}}\Pi_3, \quad \Pi_4 = \bar{G}_{\frac{1}{2}}\Pi_2 \\
\Pi_2 &= \bar{G}_{-\frac{1}{2}}\Pi_4, \quad \Pi_3 = G_{-\frac{1}{2}}\Pi_4 \\
G_{-\frac{1}{2}}\Pi_1 + L_{-1}\Pi_2 &= 0, \quad \bar{G}_{-\frac{1}{2}}\Pi_1 + \bar{L}_{-1}\Pi_3 = 0
\end{aligned} \tag{4.4}$$

The equations (3.8) show that Π_A ($A = 1, 2, 3, 4$) have the correct weight. The equations (4.4) indicate the fact that all Π_A ($= 1, 2, 3, 4$) can not depend on $\Phi^{p,q}$ with $p \geq 1$ or $q \geq 1$.

The equations (4.4) impose some conditions on Π_1 and give the possibilities to calculate Π_2 , Π_3 and Π_4 as far as we know Π_1 . In addition Π_1 must contain the terms from Beltrami parametrization [28, 7] i.e. it has to be of the form

$$\Pi_1 = X^{1,0}X^{0,1} + \dots \tag{4.5}$$

With this "boundary condition" equations (4.4) yield

$$\begin{aligned}
\Pi_1 &= X^{1,0}X^{0,1} - \lambda\lambda^{0,1} - \bar{\lambda}\bar{\lambda}^{1,0} - F^2 \\
\Pi_2 &= -\lambda X^{0,1} - iF\bar{\lambda} \\
\Pi_3 &= X^{1,0}\bar{\lambda} - \lambda\bar{\lambda} \\
\Pi_4 &= \lambda\bar{\lambda}
\end{aligned} \tag{4.6}$$

The supersymmetric invariant action can be calculated to be the term ω_2 in the local descent equation (1.2) as long as ω_0 is given by Eqs. (4.1) and (4.6). It can be, in fact, found out by projecting out in equation (2.20) the terms with ghost zero. Therefore we eventually get

$$\begin{aligned}
-2iS_{inv} &= \int dz \wedge d\bar{z} \left\{ (1 - \mu\bar{\mu}\Pi_1 + \frac{1}{2}\bar{\mu}\alpha\Pi_2 - \frac{1}{2}\mu\bar{\alpha}\Pi_3 + \frac{1}{4}\Pi_4) \right\} \\
&= \int dz \wedge d\bar{z} \left\{ \frac{1}{1 - \mu\bar{\mu}} [(\partial - \bar{\mu}\bar{\partial})X(\bar{\partial} - \mu\partial)X \right. \\
&\quad - (\alpha\lambda)(\partial - \bar{\mu}\bar{\partial})X - (\bar{\alpha}\bar{\lambda})(\bar{\partial} - \mu\partial)X + \frac{1}{2}(\alpha\lambda)(\bar{\alpha}\bar{\lambda})] \\
&\quad \left. - \lambda(\bar{\partial} - \mu\partial)\lambda - \bar{\lambda}(\partial - \bar{\mu}\bar{\partial})\bar{\lambda} - (1 - \mu\bar{\mu})F^2 \right\}
\end{aligned} \tag{4.7}$$

The action obtained by us has the same form with the one obtained by Delduc and Gieres [11].

At this point it is necessary to make the following remarks. First the solution of the equations (4.3) and (4.4) is not unique and in addition to the solution just given there is another one with Π_4 given by

$$\Pi_4 = F. \tag{4.8}$$

In this case the solution takes the form:

$$\omega_0 = -ic\bar{c} + i\epsilon\bar{c}\lambda^{0,1} - i\bar{\epsilon}c\bar{\lambda}^{1,0} + \epsilon\bar{\epsilon}F. \tag{4.9}$$

However this solution could not be used for building of an invariant action since it is linear in fields. Second, even in the case of the first solution one has a freedom in choosing the "boundary conditions". As a matter of fact Brand, Troost and Van Proeyen [29] showed that in the two dimensional gravity the most general solution of the equation (3.1) has the form

$$\omega_0 = c\bar{c}X^{1,0}X^{0,1}f(X) \quad (4.10)$$

with $f(X)$ an arbitrary function of X . We could obtain the supersymmetric extension of this solution if one starts with the following expression for Π_4

$$\Pi_4 = \lambda\bar{\lambda}f(X). \quad (4.11)$$

In this case the solution of equations (4.3) and (4.4) have the form

$$\begin{aligned} \Pi_1 &= (X^{1,0}X^{0,1} - \lambda\lambda^{0,1} - \bar{\lambda}\bar{\lambda}^{1,0} - F^2)f(X) + iF\lambda\bar{\lambda}f'(X) \\ \Pi_2 &= (-\lambda X^{0,1} - iF\bar{\lambda})f(X) \\ \Pi_3 &= (X^{1,0}\bar{\lambda} - \lambda\bar{\lambda})f(X) \\ \Pi_4 &= \lambda\bar{\lambda}f(X) \end{aligned} \quad (4.12)$$

where the prime means the derivative. In this case the supersymmetric action contains the arbitrary function $f(X)$ and it has the form

$$\begin{aligned} -2iS_{inv} &= \int dz \wedge d\bar{z} \{ (1 - \mu\bar{\mu}\Pi_1 + \frac{1}{2}\bar{\mu}\alpha\Pi_2 - \frac{1}{2}\mu\bar{\alpha}\Pi_3 + \frac{1}{4}\Pi_4) \\ &= \int dz \wedge d\bar{z} \{ \frac{1}{1 - \mu\bar{\mu}} [(\partial - \bar{\mu}\bar{\partial})X(\bar{\partial} - \mu\partial)X \\ &- (\alpha\lambda)(\partial - \bar{\mu}\bar{\partial})X - (\bar{\alpha}\bar{\lambda})(\bar{\partial} - \mu\partial)X + \frac{1}{2}(\alpha\lambda)(\bar{\alpha}\bar{\lambda})] \\ &- \lambda(\bar{\partial} - \mu\partial)\lambda - \bar{\lambda}(\partial - \bar{\mu}\bar{\partial})\bar{\lambda} - (1 - \mu\bar{\mu})F^2 \} f(X) \\ &+ i \int dz \wedge d\bar{z} F\lambda\bar{\lambda}f'(X). \end{aligned} \quad (4.13)$$

with $f'(X)$ the derivative of $f(X)$. The action (4.13) is the most general supersymmetric invariant action which depends on the matter chiral superfield with the components X , λ , $\bar{\lambda}$, F and 2D supergravity with the components μ , $\bar{\mu}$, α , $\bar{\alpha}$. It is interesting to notice that only the field X can occur in the general dependence form. Our general result can be easily extended for the matter field described by a set of chiral superfields, extension which will be given elsewhere [30].

Now we offer the results of our analyses for ghost number three. This case is very important since it leads to the possible anomalies in the super-Beltrami parametrization. The BRST cohomology for Beltrami parametrization was given in [28] (see also [19]) and it contains only four terms

$$c^{-1}c^0c^1, \bar{c}^{-1}\bar{c}^0\bar{c}^1, c^{-1}\bar{c}^{-1}c^0X^{1,0}X^{0,1}f(X), c^{-1}\bar{c}^{-1}\bar{c}^0X^{1,0}X^{0,1}f(X) \quad (4.14)$$

where $f(X)$ is an arbitrary function.

We find out the supersymmetric extensions of all these solutions. The element $c^{-1}c^0c^1$ does contain only ghosts and its supersymmetric extension must have the form

$$\omega_0 = c^{-1}c^0c^1 + ac^{-1}\epsilon^{\frac{1}{2}}\epsilon^{\frac{1}{2}} + bc^0\epsilon^{-\frac{1}{2}}\epsilon^{\frac{1}{2}} \quad (4.15)$$

being the unique combination of ghost monomials with the weight (0,0). Here a and b are two numerical constants, which can be determined from the equation (3.1). If we take into consideration the BRST transformation of c^{-1} and $\epsilon^{-\frac{1}{2}}$ and their derivatives (2.14) , (2.15) one can readily find out that $a = -2$ and $b = 1$, that is the supersymmetric extension of $c^{-1}c^0c^1$ is

$$\omega_0^1 = c^{-1}c^0c^1 - 2c^{-1}\epsilon^{\frac{1}{2}}\epsilon^{\frac{1}{2}} + c^0\epsilon^{-\frac{1}{2}}\epsilon^{\frac{1}{2}}. \quad (4.16)$$

Analogously, the supersymmetric extension of $\bar{c}^{-1}\bar{c}^0\bar{c}^1$ turns out to be

$$\omega_0^2 = \bar{c}^{-1}\bar{c}^0\bar{c}^1 - 2\bar{c}^{-1}\bar{\epsilon}^{\frac{1}{2}}\bar{\epsilon}^{\frac{1}{2}} + \bar{c}^0\bar{\epsilon}^{-\frac{1}{2}}\bar{\epsilon}^{\frac{1}{2}}. \quad (4.17)$$

The corresponding analogy can be obtained from ω_0^1 and ω_0^2 by applying the formula (1.3) and projecting out the ghost number one. In this way one gets

$$\begin{aligned} \mathcal{A}_1 &= 4 \int dz \wedge d\bar{z} (c\partial^3\mu + \epsilon\partial^2\alpha) \\ \mathcal{A}_2 &= 4 \int dz \wedge d\bar{z} (\bar{c}\bar{\partial}^3\bar{\mu} + \bar{\epsilon}\bar{\partial}^2\bar{\alpha}) \end{aligned} \quad (4.18)$$

The supersymmetric extension of $c^{-1}\bar{c}^{-1}c^0X^{1,0}X^{0,1}f(x)$ may be calculated by the same method used previously for ghost number two. We start with a solution of eq.(3.1) of the form

$$\begin{aligned} \omega_0^3 &= c^{-1}\bar{c}^{-1}c^0\Pi_1 \\ &+ \epsilon^{-\frac{1}{2}}\bar{c}^{-1}c^0\Pi_2 + c^{-1}\bar{\epsilon}^{-\frac{1}{2}}c^0\Pi_3 + c^{-1}\bar{c}\epsilon^{\frac{1}{2}}\Pi_4 \\ &+ \epsilon^{-\frac{1}{2}}\bar{\epsilon}^{-\frac{1}{2}}c^0\Pi_5 + \epsilon^{\frac{1}{2}}\bar{c}^{-1}\epsilon^{\frac{1}{2}}\Pi_6 + c^{-1}\bar{\epsilon}^{-\frac{1}{2}}\epsilon^{\frac{1}{2}}\Pi_7 + \epsilon^{-\frac{1}{2}}\bar{\epsilon}^{-\frac{1}{2}}\epsilon^{\frac{1}{2}}\Pi_8 \end{aligned} \quad (4.19)$$

The total weight of Π_A ($A = 1, 2, 3, 4, 5, 6, 7, 8$) have the values

$$\begin{aligned} w(\Pi_1) &= (1, 1) \quad , \quad w(\Pi_2) = \left(\frac{1}{2}, 1\right) \quad , \quad w(\Pi_3) = \left(1, \frac{1}{2}\right) \\ w(\Pi_4) &= \left(\frac{1}{2}, 1\right) \quad , \quad w(\Pi_5) = \left(\frac{1}{2}, \frac{1}{2}\right) \quad , \quad w(\Pi_6) = \left(\frac{1}{2}, \frac{1}{2}\right) \end{aligned} \quad (4.20)$$

The equation (3.1) yields a set of equations very similar with the ones used in the ghost case. We are not going to write them down (see [30] for details) but rather we will give only the final result. Therefore the solution of equation (3.1) of the form (4.19) is

$$\begin{aligned} \omega_0^3 &= [c^{-1}\bar{c}^{-1}c^0(X^{1,0}X^{0,1} - F^2 + \bar{\lambda}^{1,0}\bar{\lambda} - \lambda\lambda^{0,1}) \\ &+ (2\epsilon^{\frac{1}{2}}c^{-1}\bar{c}^{-1} + \epsilon^{-\frac{1}{2}}c^0\bar{c}^{\frac{1}{2}})(iF\lambda + X^{0,1}\bar{\lambda}) \\ &- \bar{\epsilon}^{-\frac{1}{2}}c^{-1}c^0(-i\lambda F + X^{1,0}\bar{\lambda}) \\ &+ \epsilon^{-\frac{1}{2}}\bar{\epsilon}^{-\frac{1}{2}}c^0\frac{1}{2}\lambda\bar{\lambda} - 2\bar{\epsilon}^{-\frac{1}{2}}\epsilon^{\frac{1}{2}}c^{-1}\lambda\bar{l}]f(X) \\ &- ic^{-1}\bar{c}^{-1}c^0F(\lambda\bar{\lambda})f'(X) \end{aligned} \quad (4.21)$$

In a similar manner one can obtain the supersymmetric extension of the $c^{-1}\bar{c}^{-1}\bar{c}^0 X^{1,0} X^{0,1} f(X)$. It has the form

$$\begin{aligned}\omega_0^4 &= c^{-1}\bar{c}^{-1}\bar{c}^0\Pi_1 \\ &+ \epsilon^{-\frac{1}{2}}\bar{c}^{-1}\bar{c}^0\Pi_2 + c^{-1}\bar{\epsilon}^{-\frac{1}{2}}\bar{c}^0\Pi_3 + c^{-1}\bar{c}\bar{\epsilon}^{\frac{1}{2}}\Pi_4 \\ &+ \epsilon^{-\frac{1}{2}}\bar{\epsilon}^{-\frac{1}{2}}\bar{c}^0\Pi_5 + \epsilon^{\frac{1}{2}}\bar{c}^{-1}\bar{\epsilon}^{\frac{1}{2}}\Pi_6 + c^{-1}\bar{c}^{-\frac{1}{2}}\bar{\epsilon}^{\frac{1}{2}}\Pi_7 + \epsilon^{-\frac{1}{2}}\bar{\epsilon}^{-\frac{1}{2}}\bar{\epsilon}^{\frac{1}{2}}\Pi_8\end{aligned}\quad (4.22)$$

with the corresponding weights for the polynomials Π_i . Performing the same algebraic calculations, one gets

$$\begin{aligned}\omega_0^4 &= [c^{-1}\bar{c}^{-1}\bar{c}^0(X^{1,0}X^{0,1} - F^2 + \bar{\lambda}^{1,0}\bar{\lambda} - \lambda\lambda^{0,1}) \\ &- (\bar{\epsilon}^{-\frac{1}{2}}c^{-1}\bar{c}^0 - 2\bar{\epsilon}^{\frac{1}{2}}\bar{c}^{-1}c^{-1})(\bar{\lambda}X^{1,0} - i\lambda F) \\ &- \epsilon^{-\frac{1}{2}}c^{-1}\bar{c}^0(\lambda X^{0,1} + iF\bar{\lambda}) + (\epsilon^{-\frac{1}{2}}\bar{\epsilon}^{-\frac{1}{2}}\bar{c}^0\lambda\bar{\lambda} - 2\epsilon^{\frac{1}{2}}\bar{\epsilon}^{\frac{1}{2}}\bar{c}^{-1}\lambda\bar{\lambda})f(X) \\ &- ic^{-1}\bar{c}^{-1}\bar{c}^0f'(X)F(\lambda\bar{\lambda})\end{aligned}\quad (4.23)$$

Using again (2.20) and projecting out the terms with ghost number 1 we can find the possible anomalies. They have the form

$$\begin{aligned}\mathcal{A}_3 &= \int dz \wedge d\bar{z} \{ [\partial\mu(\bar{\mu}c - \bar{c}) + \partial\bar{c}(1 - \mu\bar{\mu})] [(X^{1,0}X^{0,1} - F^2 + \bar{\lambda}^{1,0}\bar{\lambda} - \lambda\lambda^{0,1})] (f(X) - if'(X)F\lambda\bar{\lambda}) \\ &+ \{ [\bar{\mu}(\epsilon\partial\mu + \alpha\partial c + 2\partial\alpha(c - \mu\bar{c}) + 2\partial\epsilon(1 - \mu\bar{\mu}))](\lambda X^{0,1} + iF\bar{\lambda}) \\ &+ [\partial\mu(\bar{\alpha}c + \bar{\epsilon}) - \mu\bar{\alpha}\partial\bar{c}](-X^{1,0}\bar{\lambda} + i\lambda F) \\ &+ [-\bar{\alpha}(\alpha\partial c + \epsilon\partial\mu) + 2\mu(\bar{\epsilon}\partial\alpha + \partial\epsilon\bar{\alpha})]\lambda\bar{\lambda} \} f(X) \}\end{aligned}\quad (4.24)$$

and

$$\begin{aligned}\mathcal{A}_4 &= \int dz \wedge d\bar{z} \{ [\partial\bar{\mu}(\mu\bar{c} - c) + \partial\bar{c}(1 - \mu\bar{\mu})] [(X^{1,0}X^{0,1} - F^2 + \bar{\lambda}^{1,0}\bar{\lambda} - \lambda\lambda^{0,1})] (f(X) - if'(X)F\lambda\bar{\lambda}) \\ &+ \{ [\mu(\bar{\epsilon}\partial\bar{\mu} + \bar{\alpha}\partial\bar{c} + 2\partial\bar{\alpha}(\bar{c} - \bar{\mu}c) + 2\partial\bar{\epsilon}(1 - \mu\bar{\mu}))](\bar{\lambda}X^{1,0} - iF\lambda) \\ &+ [\partial\bar{\mu}(\alpha\bar{c} + \epsilon) - \bar{\mu}\alpha\partial\bar{c}](X^{0,1}\lambda + i\bar{\lambda}F) \\ &+ [\alpha(\bar{\alpha}\partial\bar{c} + \bar{\epsilon}\partial\bar{\mu}) + 2\bar{\mu}(\epsilon\partial\bar{\alpha} + \partial\bar{\epsilon}\alpha)]\lambda\bar{\lambda} \} f(X) \}\end{aligned}\quad (4.25)$$

As in case of ghost two we could have started the investigation of equations for Π_a with $\Pi_7 = F$. However, it seems to us that all these anomalies cannot be associated with true anomalies. Indeed, in the classical action (4.13) the terms of self-interactions are absent and in spite of the fact that these anomalies are cosmologically non-trivial, the numeric coefficients of the corresponding Feynman diagrams vanish.

5 Conclusions

We studied a part of the BRST cohomology for super-string in super-Beltrami parametrization with a given field content (super-Beltrami parametrization and Chiral matter superfield) and gave gauge invariances (superdiffeomorphism invariance). We have given a partial answer

for the cohomology not only on local functions but also on local functionals, the latter being obtained from the former by the descent equations or equivalently by using the operator δ introduced by Sorella. In this way we have obtained the most general classical action describing the superstrings in super-Beltrami parametrization given by [11], (see also [18]). Also we have found out the candidate anomalies, which are supersymmetric extension of the corresponding non-supersymmetric ones given by Brandt, Troost and Van Proeyen [29] (see also [8] and [19]). They are of two types. The first type do not depend on the matter fields and their combination provides the supersymmetric extension of the Weyl anomaly [28]. The second type anomalies depend on the matter fields and might also depend on the arbitrary functions of the scalar part X of the matter field. In the present paper the antifields and the Batalin-Vilkovisky method of quantization have not been taken into consideration. In this way we are aware that we have lost a lot of information concerning the structure and the symmetries of the model. We shall study the BV-method in the forthcoming publication [30].

Finally, we want to notice that all these calculations could be done in the superfield formulation of the theory. In this case we work only with superfields (super-Beltrami, superghosts and supermatter) and it is possible to introduce the Sorella's operator δ even for this case. We intend to give a superfield formulation of the BRST cohomology in a future publication.

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